LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2015**

MT 5506/MT 4501 - MECHANICS - I

Date : 28/09/2015 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

PART – A

Answer ALL the Questions **(10 x 2 = 20 marks)**

1. State the triangular law of forces.
2. Define like parallel forces.
3. Define moment of a force.
4. State Varignon's theorem on moments.
5. State the laws of friction.
6. Define a couple.
7. Define angular velocity.
8. Define a projectile and time of flight.
9. State Newton's experimental laws on impact.
10. Define coefficient of elasticity.

PART – B

Answer any FIVE Questions **(5 x 8 = 40 marks)**

11. State and prove Lami's theorem.
12. Two weights P and Q are suspended from a fixed point O by strings OA and OB and kept apart by a light rod AB. If the strings OA and OB make angles $\alpha \text{ and } \beta$ with the rod, show that the angle O which the rod makes with the vertical is given by $\tan\theta = \dfrac{P+Q}{Q\cot\beta - P\tan\alpha}$.
13. Two like parallel forces P and Q (P>Q) act at A and B respectively. If the magnitudes of the forces are interchanges, show that the point of application of the resultant on AB will be displaced through the distance $\dfrac{P-Q}{P+Q}.AB$.
14. Two rough particles connected by a light string rest on an inclined plane. If their weights and corresponding coefficients of friction are W_1, W_2 and μ_1, μ_2 respectively and $u_1 > \tan\alpha > u_2$, where α is the inclination of the plane with the horizon, prove that $\tan\alpha = \dfrac{\mu_1 W_1 + \mu_2 W_2}{W_1 + W_2}$, if both particles are on the point of moving down the plane.
15. If A and B describe concentric circles of radii a and b with speeds u and v, the motion being the same way round. If the angular velocity of either with respect to the other is zero, prove that the line joining them subtends at the centre and angle whose cosine is $\dfrac{au+bv}{av+bu}$.

16. The speed of a train increases at a constant rate α form O to v and then remains constant for an interval and finally decreases to zero at a constant rate β. If d be the total distance covered, prove that the total time occupied is $\frac{d}{v}+\frac{v}{2}\left[\frac{1}{\alpha}+\frac{1}{\beta}\right]$.

17. Show that when masses P and Q are connected by a string over the edge of a table, the tension is the same whether P hangs and Q is on the table or Q hangs and P is on the table.

18. A ball impinges on another equal ball moving with the same speed in a direction perpendicular to its own, the line joining the centres of the balls at the instant of impact being perpendicular to the direction of motion of the second ball. If e is the coefficient of restitution, show that the direction of motion of the second ball is turned thorugh $\tan^{-1}\left(\frac{1+e}{2}\right)$.

PART – C

Answer any TWO questions **(2 x 20 = 40 marks)**

19. a) Three equal strings of no sensible weight are knotted together for form an equilateral Δ ABC and a weight w is suspended from A. If the triangle and the weight be supported with BC horizontal by means of two strings at B and C each at angle 135° with BC, show that the tension in BC is $\frac{W}{6}(3-\sqrt{3})$. (10)

b) A uniform rod A B of length 2a and weight W is resting on two pegs C and D in the same level at a distance d apart. The greatest weights that can be placed at A and B without tilting the rod are W_1 and W_2 respectively. Show that $\frac{W_1}{W+W_1}+\frac{W_2}{W+W_2}=\frac{d}{a}$. (10)

20. a) A system of forces in the plane of Δ ABC is equivalent to a single force at A, acting along the internal bisector of the angle BAC and a couple of moment G_1. If the moments of the system about B and C are respectively G_2 and G_3, Prove that $(b+c)\,G_1 = bG_2 + c\,G_3$. (10)

b) A body, sliding down a smooth inclined plane, is observed to cover equal distances each equal to a, in consecutive intervals of time t_1, t_2. Show that the inclination of plane to the horizon is $\sin^{-1}\left[\frac{2a(t_1-t_2)}{gt_1t_2(t_1+t_2)}\right]$. (10)

21. a) Two particles of masses m_1 and m_2 ($m_1>m_2$) are connected by means of a light inextensible string passing over a light, smooth, fixed pulley. Discuss the motion. (10)

b) A particle falls under gravity in a medium where resistance varies as the velocity. Discuss the motion. (10)

22. a) Derive the equation to the path of a projectile. (12)

b) A particle is to be projected from a point so as to pass through another point Q. Show that the product of the two times of flight from p to Q with a given velocity of projection is $\frac{2}{g}$PQ. (8)

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